# Math 4550 - Homework # 5 - Cyclic Groups

## Part 1 - Computations

1. Find all the subgroups of $\mathbb{Z}_8$.

2. Find all the subgroups of $U_6$.

3. Suppose that $\phi : U_5 \to \mathbb{Z}_5$ is a homomorphism with $\phi(e^{2\pi i/5}) = \overline{2}$. Find all the values of $\phi$ and draw a picture of $\phi$.

4. Find all homomorphisms $\phi : \mathbb{Z}_4 \to \mathbb{Z}_6$.

5. Find all homomorphisms $\phi : U_3 \to \mathbb{Z}_6$.

6. Find all homomorphisms $\phi : U_5 \to \mathbb{Z}_4$.

7. Find all homomorphisms $\phi : \mathbb{Z}_6 \to \mathbb{Z}_3$.

8. Find all homomorphisms $\phi : \mathbb{Z} \to \mathbb{Z}_3$.

9. Suppose that $\phi : \mathbb{Z} \to \mathbb{Z}$ is a homomorphism with $\phi(1) = 5$. Find a formula for $\phi(n)$ for all $n \in \mathbb{Z}$.

## Part 2 - Proofs

10. Let $G$ be a group and $x \in G$.

    (a) Prove that $\langle x^{-1} \rangle = \langle x \rangle$.
    (b) Prove that $x$ and $x^{-1}$ have the same order.

11. Here are some examples of infinite abelian groups that are not cyclic.

    (a) Prove that the group of rational numbers $\mathbb{Q}$ under addition is not cyclic.
    (b) Prove that the group of real numbers $\mathbb{R}$ under addition is not cyclic.

12. Let $\phi_n : \mathbb{Z} \to \mathbb{Z}_n$ be given by $\phi_n(x) = \overline{x}$.

    (a) Draw a picture of $\phi_3$.

    The following are for general $n$:
    (b) Prove that $\phi_n$ is a homomorphism.
    (c) Prove that $\phi_n$ is onto but not one-to-one.
    (d) Prove that $\ker(\phi_n) = n\mathbb{Z}$.